

Persistence as Order Parameter in Generalized Pair Contact Process with Diffusion

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Abstract. The question of universality class of pair contact process with diffusion (PCPD) is revisited with an alternative approach. We study persistence in Generalized Pair-Contact Process with diffusion (GPCPD) introduced by Noh and Park, (Phys. Rev. E 69,016122(2004)). This model allows us to interpolate between directed percolation (DP) and PCPD universality classes. We find that transition to nonzero persistence is at same parameter value as transition to zero number density. We obtain finite size scaling and off-critical scaling collapse for persistence and find critical exponents by fitting phenomenological scaling laws to persistence. While the dynamic scaling exponent z varies continuously in GPCPD, the correlation-time exponent ν_{\parallel} matches with directed percolation universality class.

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1. Introduction

Phase transitions in non-equilibrium systems have been investigated in detail in past two decades. One of the earliest, most extensively studied and most commonly observed non-equilibrium transition is the transition to an absorbing state. It is often described by Directed percolation (DP) universality class. Grassberger and la Torre established equivalence of Reggeon Field theory with Markov process. This was followed by Janssen and Grassberger's conjecture that all transitions to nondegenerate absorbing state in one-component systems without quenched disorder, with short range interactions in space and time, and in absence of multi-critical points are in DP universality class. By now, we know several systems in this universality class. [1, 2]

There are very few systems which are unambiguously shown to be in an universality class different from DP. Systems such as branching and annihilating random walks with even number of offsprings (BAWe) [3], model A and B of probabilistic cellular automata[4], interacting monomer-dimer models [5] and nonequilibrium kinetic Ising model in presence of spin flip as well as spin exchange dynamics [6] are found to be in directed Ising (DI) universality class[7]. Voter universality class, parity conserving class (which has same exponents as DI in 1-d) and absorbing phase transitions with conserved field are such examples[8]. Compact directed percolation is another such class[9]. Models such as Manna model were considered to be in different universality class. However, questions have been raised on the independent Manna class[10]. Special boundary conditions, quenched disorder and anomalous diffusion affect the exponents[11]. In this work, we study a process known as pair contact process with diffusion (PCPD) which has been reported to be possibly in an independent universality class using an alternative order parameter.

A model with infinite absorbing states called pair contact process (PCP) did attract lot of attention. In PCP, particles in two neighboring occupied sites can annihilate each other with probability p and produce an offspring with probability $1 - p$. Any state without pairs is an absorbing state. Infinity of such states are possible. Despite not obeying sufficient conditions for DP universality class, this transition is considered to be in DP universality class[12].

A variant of PCP known as pair contact process with diffusion (PCPD) attracted even more attention. The universality class of this transition in 1-d has been a matter of long-standing debate and the issue is not yet settled. (for further generalization, see [13].) Initially, it was seen as process in a distinctly new universality class [14]. But, several alternatives were proposed in later studies on this model (See Sec. I of [15]).

One of the most plausible arguments for PCPD being in DP class was given by Hinrichsen. He argued that isolated particles can spread at most diffusively with dynamic exponent 2. But most of the estimates of dynamic exponent of PCPD yield value less than 2 implying a superdiffusive spread of critical clusters. Hence the asymptotic behaviour should not be dominated by diffusion of isolated particles which are effectively frozen. If the isolated particles are frozen PCPD reduces to PCP which belongs to DP universality class[16].

The model shows very strong corrections to scaling. In such cases, it is argued that ‘very large scale and long-time computations in future will help us to settle on an appropriate conclusion’. This approach of longer and longer simulations on larger and larger systems has been addressed to some extent in recent decade and simulation time has reached 10^9 recent paper by Park[17]. He has concluded that on including corrections to scaling, the critical density decay exponent is 0.173 independent of diffusion constant. Thus this model is incompatible with DP universality class. Barkema and coworkers argue that the model is in same universality class as DP by considering scaling of ratio of pair density and particle density[18, 19] and assuming certain corrections to scaling. Due to strong corrections to scaling, there is a debate even about critical density decay

exponent. On the theoretical side, this problem has resisted analysis and the latest work by Gredat *et al* in which PCPD has been studied by nonperturbative functional renormalization group has suggested that ‘critical behavior of PCPD can be either in DP or a in a new (conjugated) class’[20]. The numerical studies on scaling of order parameter and theoretical studies have not reached a conclusive end. Hence, there is a need for an alternative approach. We work on a model which reaches PCPD in certain limit and DP in another limit.

Critical density decay exponent δ for DP is known to be 0.1595 while that for PCPD has ranged from 0.17-0.275 [19] and the latest estimate is as low as 0.17 [17]. Thus the difference between exponents (if any) is very small. The value of exponents is very small as well. It takes very long for system to settle into saturation state and time grows with system size. The static exponents which differ in magnitude much more are difficult to compute. For example, if we work on modest system size of 5×10^4 lattice sites, it will need 10^{30} time steps before the number density goes to zero at critical point. Thus for a small δ , very long simulations become necessary at large sizes. In presence of strong corrections to scaling, even these simulations do not give reliable results. Other parameters such as pair density decay in a similar manner and they can help in correcting the corrections to scaling. We attempt an alternative order parameter which decays much faster. Unfortunately, a larger decay exponent brings in its own difficulties such as larger relative error and difficulty in computing static exponents.

In this work, we attempt to find indications of universality and scaling behavior from short-time behavior check for alternative ‘order parameter’. Recently, there have been attempts to quench the system from high temperature to critical temperature, study its short time dynamics and analyze its relation with the universality class[21, 22]. There have been some successful attempts at obtaining critical exponents from scaling behavior of persistence[23, 24, 25] which is again short-time history dependent behavior. We study persistence in PCPD and its generalized version from this point of view.

2. Persistence

Persistence is a generalization of first passage time for spatially extended systems. In spin systems, local persistence at time T , $P(T)$ is the fraction of sites which did not change their initial spin state at all times $t \leq T$. In Ising type systems, it may show power law decay in time at zero temperature and exponent is called persistence exponent. In contact processes, persistence may show power law at critical point and the exponent is called persistence exponent[26].

The persistence exponents themselves are not universal and depend on delicate details of evolution. Models falling in same universality class have the same persistence exponent in some cases, but not necessarily so. For example, persistence exponent of widely different models such as 1-d Ising model, coupled logistic maps and Sznajid model is $\frac{3}{8}$ [23, 27, 28]. Similarly, it has been observed the persistence exponent in 1-

D DP models such as Domany kinzel model[29], Ziff-Gulari-Barshad model [30], site percolation[25, 31] and 1-d coupled circle maps [32] is same, *i.e.* $\frac{3}{2}$ or very close to it. In Ref. [30], they carried out work on 2-d DP model and claimed that persistence exponent is superuniversal. Now there is reasonable evidence that persistence exponent is not even universal[33]. Its value may be of interest from the viewpoint of detailed dynamics. We argue that even if persistence exponent is not universal, it can help in finding other universal exponents.

This nontrivial exponent sheds further light on detailed dynamical nature of phase transitions [23]. Furthermore, its scaling behavior also sheds light on other exponents in the system. Some examples are as follows a) Fuchs *et al* studied 1+1 dimensional contact process in DP class, and obtained a successful data collapse of persistence for finite size scaling as well as off-critical simulations using 1-d DP values of $\nu_{||}$ and z [25]. b) The onset of spatiotemporal intermittency in coupled circle maps is known to be in DP universality class. Finite size scaling of persistence yields estimate of z which matches with 1-d DP value[32]. c) Hinrichsen and Koduvely studied persistence in Domany-Kinzel automata and found that persistence obeys finite size scaling with same exponents as DP[29]. d) An evolutionary model of Prisoner's dilemma on 2-d lattice is found to show a transition in DP universality class. Two variants of this model have been studied and $\nu_{||}$ obtained from off critical simulations of persistence in this model match with 2-d DP values[33]. e) Manoj and Ray obtained finite-size scaling collapse of persistence in Glauber-Ising model in dimensions 1-4 [34] at zero temperature. Thus there are few cases where transition is known and the scaling collapse for persistence is obtained by using standard values.

We try this approach towards this problem and try to complete the study. Local persistence in this model is not studied before. We study a model called GPCPD (Generalized Pair Contact Process with Diffusion) which smoothly interpolates between DP and PCPD with a parameter controlling memory strength [35]. We compute the persistence exponent for this model which is a new nontrivial critical exponent unrelated to other critical exponents.

Apart from completing studies by determining persistence exponent, the studies can be used for validation of other critical exponents, particularly when studies on order parameter have not been conclusive. As mentioned above, there have been four prior cases in which persistence in 1-d and 2-d DP models has been studied and finite size scaling and off-critical scaling in these models give an estimate of z and $\nu_{||}$ which agrees with standard DP models. We observe a fairly convincing scaling in GPCPD using this approach and determine scaling exponents in this manner. In this model, for limiting case of DP, we indeed get scaling exponents close to those for DP. Thus it is fair to expect that if PCPD were in DP class, DP scaling for persistence should prevail in finite size scaling as well as off-critical simulations. Our basic result is that while z varies continuously as reported by Noh and Park, the correlation-time exponent $\nu_{||}$ matches with DP for GPCPD.

3. The model

Noh and Park introduced an extra free parameter r controlling memory. In this model, if pair of occupied sites is chosen, it can either lead to creation of one more particle with some probability, or both particles can be annihilated with certain probability. If this was the only dynamics, the state with no pairs is an absorbing state and such a transition is known to be in DP class. However, picture changes if we allow particles to diffuse. For nonzero diffusion, we have two absorbing states. Apart from vacuum state, we have a state with single diffusing particle. Such state is not unique. It can be termed as L dimensional absorbing subspace to which particle is confined.

The pair density in this model can change due to diffusion which leads to long-term memory. The pair created at some time due to diffusion can proliferate and lead to extra particle after a very long time. The parameter r controls creation of diffusion-induced pairs. The simulation is carried out in following manner. Consider a 1-d lattice of N sites with periodic boundary conditions. A pair of sites i and $i + 1$ is randomly selected. If both sites occupied, the particles are annihilated with probability $(1 - d)p$ or a particle is added to a neighboring $i - 1$ or $i + 2$ 'th site with probability $(1 - d)(1 - p)$ provided neighboring site is empty. If only one of the i and $i + 1$ 'th site is occupied, the occupied particle hops to other site with probability d . If hopping solitary particle creates a pair by coming in contact with site occupied by a particle, both particles are annihilated with probability $1 - r$ as mentioned before. One time-step is completed after N such trials. In short, if diffusion leads to formation of particle pair, such pair is annihilated with probability $1 - r$. For $r = 0$, such pair is certainly annihilated and diffusion does not increase number of particle pairs. For $r = 0$, when the system reaches an absorbing state of PCP, *i.e.* there are no consecutive occupied sites, the dynamics thereafter is just diffusion. If this diffusion leads to particle pairs the particles are annihilated and the final absorbing state can be a vacuum state or a solitary particle. The number of absorbing states is not infinite for $r = 0$ case as it is for $d = 0$ case. But the absorbing state for $r = 0$ case is same as that for PCP since the dynamics thereafter is plain diffusion. In this case, it is reasonable to expect that the transition is in DP class [12, 35]. For $r = 1$, diffusion occurs independently and can change the number of particle pairs. This case is equivalent to PCPD. Noh and Park relate the additional parameter r introduced by them to memory since if diffusing particles can lead to particle pairs, pair-creation rate will be history dependent. This history dependence will not be there if the diffusing particles certainly annihilate on meeting.

Noh and Park have reported accurate critical points at which transition is observed for $d = 0.1$. The values are $p_c = 0.046872, 0.055055, 0.066364, 0.083155$ and 0.1112 for $r = 0, 0.25, 0.5, 0.75$ and $r = 1$. For $r = 1$ an improved value 0.111158 has been reported by Park[17]. They investigated pair density and systematically found values of all exponents for various values of r . They found continuously varying exponent for various values of r . We are not aware of any further theoretical or numerical studies in GPCPD after Noh and Park's work.

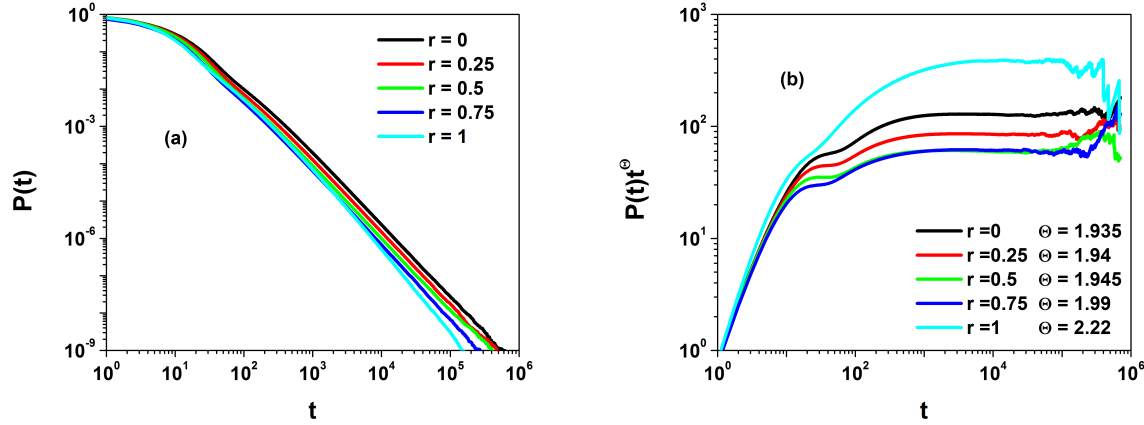


Figure 1. a) $P(t)$ as a function of time t , for various values of r at critical point. We carry out simulation for $N=3 - 5 \times 10^5$ sites and average over 2×10^5 configurations. b) $P(t)t^\theta$ as a function of time t , for various values of r at the critical point for same data as (a).

4. Results

We investigate this problem using persistence as ‘order parameter’. We define persistent sites at time T as ones which did not change their initial state at all time steps $t \leq T$. We find that the asymptotic persistence is zero for $p < p_c$. The p_c values at which persistence goes to zero match with p_c values mentioned above. We obtain excellent power-laws for our order parameter decay for GPCPD at these points.

We have simulated the GPCPD model for lattice size of $N = 2 \times 10^5$ for $r = 0, 0.25, 0.5, 0.75$ and $r = 1$ and $d = 0.1$ and average over more than $3 - 5 \times 10^5$ configurations. In Fig. 1a), we plot fraction of persistent sites, $P(t)$ as a function of time t for these values of r at the critical point which exhibits a clear power-law decay. The exponent obtained ranges between 1.935-2.22 and it increases with r . If $P(t) \sim t^{-\theta}$, $P(t)t^\theta$ should be a constant. We have also plotted $P(t)t^\theta$ as a function of t in Fig. 1b), and it shows a flat line asymptotically. We observe a flat line over several decades for $r \neq 1$. Unfortunately, for $r = 1$ the onset of scaling is late and it is observed over fewer decades.

We use well established phenomenological scaling laws extended to persistence[25, 36]. For system size N and the distance from criticality $\Delta = |p - p_c|$, We expect following asymptotic law to hold

$$P_N(t) = t^{-\theta} F(t/N^z, t\Delta^{\nu_\parallel}) \quad (1)$$

where F is the scaling function and $z = \frac{\nu_\parallel}{\nu_\perp}$ is dynamic scaling exponent. Though θ is not an universal exponent, z and ν_\parallel are. We hope that these scaling relations will shed light on possible universality class of underlying models.

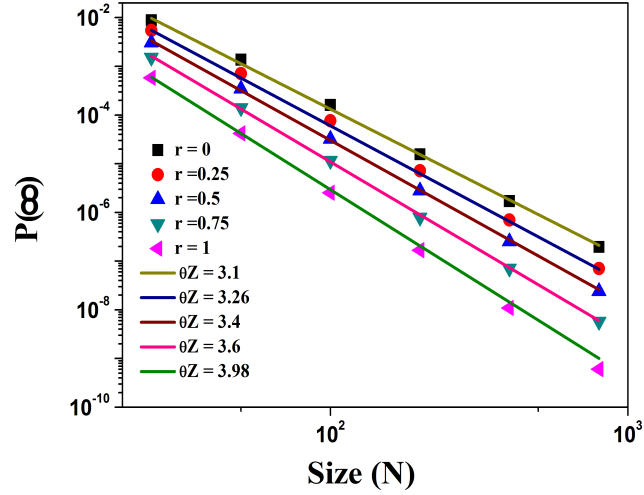


Figure 2. a) Saturation value of Persistence $P(\infty)$ as a function of N is plotted for various value of r at $p = p_c$. We wait for $10^8 - 10^{10}$ time-steps and average over $6 \times 10^5 - 10^7$ initial conditions with longer waiting period and higher averaging for larger N .

We consider two special cases.

a) Finite-size scaling : We find the persistence probability at the critical points $p = p_c$, for various values of r for finite sizes. We define $\Delta = |p - p_c|$. Thus we conduct simulations for $\Delta = 0$ for various values of r . Thus we expect, $P_N(t) \simeq t^{-\theta} F(\frac{t}{N^z})$.

Like order parameter, the persistence probability saturates to some value for $t > N^z$. The saturation value for $P(t)$ is expected to be proportional to $N^{-z\theta}$. Thus plotting $P(t)N^{z\theta}$ as a function of t/N^z is expected to yield a good scaling behavior. In Fig. 2, we have plotted saturation value of persistence $P(\infty)$ at critical point for various values of N . The slope is expected to yield value of $z\theta$ which increases monotonically with r and the values of z are not compatible with DP for these sizes.

We have plotted $P(t)N^{z\theta}$ as a function of t/N^z at $p = p_c$ for various values of r shown in Fig. 3. A good scaling behavior is obtained for values of z obtained from $z\theta$ values inferred from saturation value of persistence in Fig.2. A successful scaling collapse is obtained for all values of r . The value of $z = 1.60$ for $r = 0$ is close to DP exponent $z = 1.581$. For $r > 0$, our values of z increase with r as reported in [35]. The precise values are slightly larger than those reported above. The value z for $r = 1$ is in agreement Noh and Park's estimate from static simulations and there is no report of $z < 1.8$ from static simulations in earlier works. This value is also in agreement with recent estimates from static simulations without assuming any corrections to scaling. (See table II of [19]) The values of z reported in literature decrease with d and approach DP as $d \rightarrow 1$ (See table III of [15]).

The value of θ changes slowly for smaller values of r and shows a big jump at $r = 1$. This change is not reflected in z which saturates for large value of z .

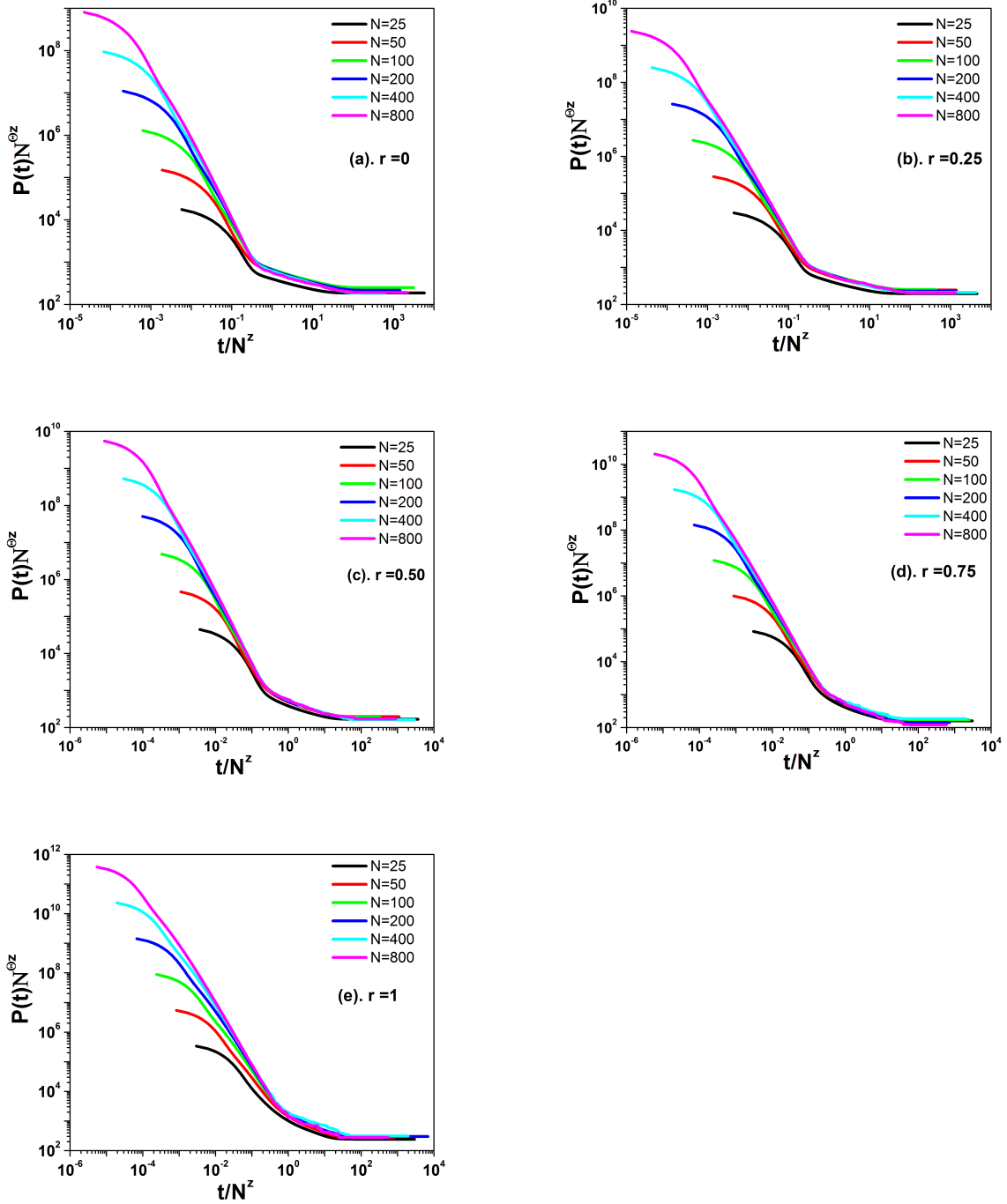


Figure 3. Persistence $P(t)N^{z\theta}$ as a function of t/N^z for various values of N at critical point $p = p_c$. We average over $10^5 - 10^6$ configurations with more averaging for larger values of N .

Obtaining good statistics for saturation values of $P(t)$ for larger N is made difficult due to the fact that $z\theta \gg 1$. If $z\theta = 1$ (say), and 4% sites were persistent for $N = 100$, 1% sites will be persistent for $N = 400$ and number of persistent sites for $N = 400$

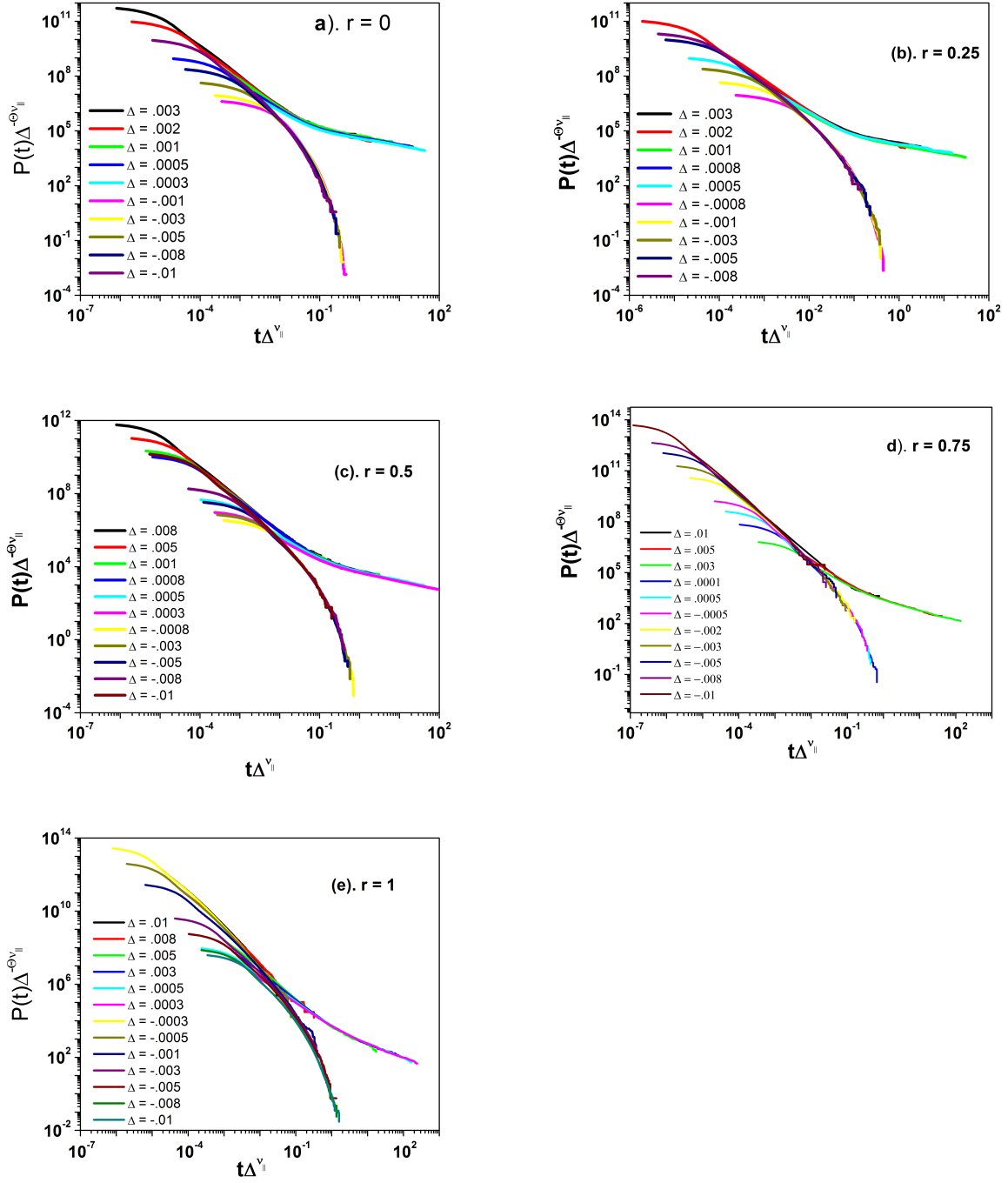


Figure 4. Persistence $P(t)\Delta^{-\theta\nu_{||}}$ is plotted as a function of $t\Delta^{\nu_{||}}$ for various values of $\Delta = |p - p_c|$. The limiting curves are different for $p > p_c$ and $p < p_c$. We average over 5×10^4 configurations for $p < p_c$ and $5 \times 10^3 - 10^4$ configurations for $p > p_c$.

will be 4 again. The expected *number* of persistent sites is actually same for single configuration. It is easy to see that the number of configurations required to have reliable statistics grows like $N^{z\theta-1}$. If $z\theta < 1$, less averaging is required for large N ,

Table 1. Critical exponents for GPCPD and comparison with Ref. [35]

r	$\nu_{ }$ [35]	$\nu_{ }$	z [35]	z	θ
0	1.738	1.73	1.58	1.60	1.935
0.25	1.80	1.73	1.64	1.68	1.94
0.5	1.86	1.73	1.69	1.74	1.945
0.75	2.01	1.73	1.72	1.80	1.99
1	2.34	1.73	1.8	1.81	2.22

Table 2. Comparison of DP, PCPD and DI

Class	$\nu_{ }$	z
DP [35]	1.73	1.58
PCPD (d=0.1) [15, 19]	1.61- 2.45	1.8- 2.08
DI [35]	3.22	1.75

For $z\theta > 1$, a single configuration of with large N will have no persistent site at critical point asymptotically. Apart from the fact that we have to wait for longer time for larger values of N , much higher averaging is required to obtain reliable statistics. Hence we have not carried simulations for larger sizes. Another reason is that uncertainty in critical point would lead to significant errors in value of z for large N . Overestimate of p_c will lead to underestimate of value of z and vice versa.

b) Off-critical simulations: We conduct simulations for $N = 2 \times 10^5$ and average over at least 8×10^3 initial conditions for $p < p_c$ and at least 10^3 initial conditions for $p > p_c$. The value of $z\theta$ ranges from 3.1 – 3.98 and saturation value at critical point for $N = 10^4$ will be of order $N^{-z\theta} \sim 10^{-12}$ or less and below numerical precision. For all practical purposes, finite size corrections can be ignored. The scaling form mentioned above indicates a standard scaling collapse when we plot $P(t)\Delta^{-\theta\nu_{||}}$ as a function of $t\Delta^{\nu_{||}}$ where $\Delta = |p - p_c|$. The persistence saturates for $p > p_c$ and goes to zero for $p < p_c$ which is exactly opposite to the behavior of particle density or pair density. We carry out this scaling for various values of r and find value of $\nu_{||}$. In Fig. 4, we have shown scaling collapse for various values of r and Δ . The best fitting values of $\nu_{||}$ are given in table below. For $\nu_{||} = 1.73$, we obtain excellent scaling collapse for $r = 0, 0.25, 0.50, 0.75$ and 1. This value of $\nu_{||}$ matches with that of DP.

5. Discussion

We have found that in GPCPD a) Persistence decays as power law at critical point. b) The persistence exponent is a function of r and c) The persistence exponent is much larger than δ and presents a better scaling behaviour. Thus it can be used to find other exponents such as z and $\nu_{||}$. Of course, all three critical exponents cannot be found by this approach (since $\delta \neq \theta$). However, it is not necessary that there will be a well defined

persistence exponent at the critical point, or that the persistence exponent will be larger than δ . Only argument we can give for validity of phenomenological scaling laws is cases in literature where it has been successfully used. It is necessary to study the scaling of persistence and validity of various scaling laws in cases where the universality class of model is well established. This will enable this approach is placed on a firm footing, at least numerically.

Our values of z are broadly in agreement with Noh and Park. We have not been able to go for very large sizes for computation of z since the saturation value of persistence even for $N = 400$ or 800 is reached only after 10^9 timesteps or more. This makes it very difficult to go to larger sizes. It is very unlikely that the static exponent ν_\perp changes with r and ν_\parallel does not change. We believe that simulations for larger sizes and longer times will yield value of z which is compatible with DP.

Noh and Park speculate, that their model has continuously varying set of exponents. Given the broad range of exponents obtained in literature for PCPD, there was a speculation that PCPD has continuously varying set of exponents[15]. In Table 3 of ref. [15], several values of critical exponents for 1D PCPD for $d = 0.1$ from literature are listed. The values of z range from 1.8 [35] to 2.04 [37]. The values of ν_\parallel can be obtained $\nu_\parallel = z\nu_\perp = \frac{z\beta}{\nu_\perp}$ and they range from 1.61 [38] to 2.45[14]. Needless to say, there is even further variation with d and authors generally report smaller values of z for larger d . It has been proposed that PCPD has continuously varying set of exponents depending on value of d [37]. Noh and Park conjectured that it is possible that GPCPD is in same class with further variation due to extra free parameter r . However, the idea of continuously varying exponents is not very compatible with classical idea of universality which leads to few exponents depending on dimensionality and symmetries[20]. Recent works in PCPD have suggested a clear drift of exponents toward DP values for long-time simulations.

Our work suggests that the value of ν_\parallel in GPCPD is same as in DP. However, our value of z differs considerably from DP values. This is a feature shared by analysis of all static simulations in PCPD without assuming power-law or logarithmic correction. This departure from DP values could be an artifact of finite size and finite time simulations. In a cellular automata model analogous to PCPD, Hinrichsen has clearly demonstrated that the values of z reduce as a function of t [39] and static simulations may not be the best way to obtain dynamic exponent z .

We have revisited the problem of universality class of PCPD with an alternative approach. Despite extensive numerical effort, we are not able to completely settle this issue. However, above results indicate that we cannot rule out the possibility that PCPD is in same universality class as DP.

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